## Dimensions of the Universe - I

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## Fundamental Questions - I

The first, base, question considered here is: What is the distance between two points?
At first sight, this may seem to be a simple question to answer - but actually there are complications in the real world.

If the two points under consideration are, for example, points on the same circle, then there are three possible ways of expressing the distance between them (see Figure 1):

- as the arc distance along the circle (arc AB), or
- as the chord distance across the circle (chord AB ), or
- as the angle subtended between the two points from the centre $(\theta)$

These three possible measures are expressed in different units: both the arc distance and the chord distance may be expressed in units of length, but the angular distance is expressed as an angle - and therefore is not, strictly speaking, a length - though if the radius of the circle is also known and given, then we have a length expressed as a pair $(r, \theta)$.


Figure 1.

[^0]
## Thought Experiment

Consider a curve - for the moment take it to be an arc of a circle, but the exact shape is something to be reconsidered later. The circle is expanding over time, at (first assumption) a constant rate. That is, the radius of the circle is growing at a constant rate over time - the linear amount (not proportional amount) of increase of the radius is the same in each unit time. Consider also two points, A and B, on initial (inner) circle, which project outwards to the points A' and B' on an outer circle (see Figure 2):


Figure 2.
Here there are more options for the distance between A and B: we have, as before,

- the arc distance along the inner circle (arc $A B$ ), and
- the chord distance across the inner circle (chord AB ), and
- the angle subtended from the centre, with radius $(r, \theta)$

But we also have the same three for the outer circle:

- arc A'B', and
- chord A'B', and
- angle with radius ( $r+a, \theta$ )

Taking this diagram as an image of the universe expanding, then there is one further possibility to consider (see Figure 3):


Figure 3.

If the circle is a mapping of just one space dimension (along the arc), with the radius being a mapping of Time $(t)$ then any given point on the circle is an event, with spatial location (its place on the circle) and a time coordinate (the then radius of that circle). As time continually increases, this radius also increases. Note that this brings in an absolute time measure - something which is anathema within Relativity, and which we must (and will) remove.

In the real universe, we can measure distance by sending a light-signal between the points under consideration. In this diagram that would be along a line joining A and B'. So yet another measure of the distance between the original points is:

- the length of the line AB' (or AD in Figure 4).

Strictly speaking, this is not a line between two points at the same time, but between two events, each with its own space-time location. Again, we are viewing this "from the outside", assuming that we can meaningfully speak of an absolute space reference, and an absolute time reference which applies to both of these points from the original (inner circle) time right up to the ending (outer circle) time. And absolute space is something else we shall consider later - and remove from the final image.

We are now considering, in this image, three dimensions - not fully independent of each other. The first dimension is in the radius direction, pointing outwards from the centre of the circle, and crossing the arc at right angles. For this description, this thought-experiment, we are calling this dimension Time [T], but note that as it is measured in units of length we shall call it Direction(T) [DT]. The second dimension is at right angles to this, and points along the tangent to the (circular) arc. For this description we shall call this Distance(S) [Ds]. This too is measured in units of distance. The third dimension - clearly not independent of the other two - is measured in distance units along the (circular) arc. For this description we shall call this Distance(A) [DA]. See Figure 4.


Figure 4.
The distance between points A and B may be measured in the $T$ direction ( D ), and (from A to B ) is positive. Call this $\alpha \times t$ units, where $\alpha$ is the radial speed of expansion and $t$ is time.

## The Expanding Universe

We know that the universe is expanding [1]. One suggestion is that it is expanding at the speed of light - but no particular assumption like that is being made here. We are assuming the speed of expansion is $\alpha$ units of distance per second (unit of time). The space into which it is expanding, however, is not the normal space that we experience - it is, in some sense, orthogonal to the dimensions that we normally experience. That is, though the expansion is experienced in all of the (spatial) dimensions we can sense, the expansion is taking place in at least one other dimension.

So how, within this expanding universe, do we measure distance? Along Dc or Ds or DA or DT? Or something else? ${ }^{2}$

The distance between two points on an arc may be measured in several different ways. One is to measure the distance in units of DA , along the single arc. This is the arc distance A to C (see Figure 3 ). If this is the choice made, then (in the simplest mapping form this thought experiment to the actual universe) the rate of expansion of this arc directly relates to the Hubble Constant.

Another, completely different, way is to measure the distance in units of Ds along the tangent to the inner-most arc, until that tangent meets the point in its expanded position on the outer-most arc. This is the line distance A to D in Figure 4, $f$.

A third way to measure distance is along the chord AB , on a line which lies outside of the notional "space" (the circumference of the circle) and which touches that space only at the two points. This is the direction Dc.

A fourth way to measure the distance is to measure it in units of T, along the radius, to the matching point on the outer-most arc to the starting point, when the tangent from the inner-most arc meets the outer-most arc. This is the distance from A to G in the diagram (Figure 1), $t$.

If we express the arc distance in terms of the speed of light, then it is $c \times t$. Similarly, the tangential distance ...

We have:

$$
\begin{gathered}
b=r \theta=c \times t \\
f=r \tan (\theta) \\
\cos (\theta)=r /((\alpha \times t)+r) \\
t=f \tan (\theta)=r \tan ^{2}(\theta)
\end{gathered}
$$

Since (in this thought-experiment) we know $a$ and $t$, we can compute $\theta$ from $\tan ^{2}(\theta)=t / r$. Note that for $\theta<\pi / 4$ we have $\tan (\theta)<1$ and also $f>b$ - that is, the "straight line" distance AF ( $f$ ) is longer than the "arc" distance AC (b).

If we consider two points $A$ and $D$ such that the angle AOD is greater than $45^{\circ}$ (or $\pi / 4$ ) but less than $\pi / 2\left(90^{\circ}\right)$ then the "straight line" distance may be very much greater than the "arc" distance, and even greater than the radius distance from O of A ( $a$ ). This "straight line" distance approaches infinity as $\theta$ approaches $\pi / 2$.

[^1]

Figure 5
To give some feel of the magnitudes involved here, at present the observable universe is about 13.8 thousand million years old, or 4.35 E 17 seconds. The speed of light is about 2.997 E 8 metres per second ${ }^{3}$, hence if the universe is expanding at the speed of light [another assumption - not one we shall sustain] the time-radius of the observable universe is at least $4.35 \mathrm{E} 17 \times 2.997 \mathrm{E} 8$ metres, or about 1.304E26 metres. Other calculations suggest that the observable radius is 4.3 E 26 metres. The value $a$ is one of (or between) these two values.

In this thought-experiment the rate of expansion of the universe in the T direction is related to the speed of light, or (more strictly) to $c$ - the notional speed of light in vacuum. What that relationship is, though, has yet to be made clear. So the initial assumption being made is that the local speed of light (c) is defined by the local rate of expansion of the universe. For the moment this leaves open the question of whether $c$ is constant across the (spatial) universe and across time. Note that we are not saying that $\quad \alpha$ (the rate of expansion) is the same as $c$ (the speed of light) but is related to it.

If, for simplicity, we at first examine the simplest relationship, which is the case where $\quad \alpha=c-$ that is, they are equal - then if $\theta$ is only one arcsecond ( $0^{\circ} 0^{\prime} 1^{\prime \prime}$ ) the distance between A and C (or A and F ) is between 6.32E21 and 2.08E22 metres. Up to an angle of seven arcseconds ( $0^{\circ} 0^{\prime} 77^{\prime \prime}$ ) the difference between the arc AC and the line AF is less than one part in 1E11 - and that allows A and C to be between 4.42E22 and 1.46E23 metres (depending on the value of $a$ that we take). For scale, the Andromeda galaxy is about 2.4E22 metres away from the Solar System (by current estimates).

In the extreme case, if the angle AOD is $90^{\circ}$ (or $\pi / 2$ ) then D is unobservable from A . And that is true for the whole semicircle where $\theta \geq \pi / 2$ - half of the possible universe. There are further reasons (considered later) that reduce even further the observable proportion of the universe.

[^2]
## Measuring Distance

We have labelled the OAG direction with the letter $t$, and in this thought-experiment that is time. The different angles to the radius represent different speeds - along the radius it is zero speed (with reference to the originating event), and along the tangent, at right-angles to the radius, it is the limit speed, $c$, the (notional) speed of light in vacuum (with reference to all events through which the tangent temporarily passes as tangent). If we think of sending a light signal from A to C in this thought experiment the signal arrives at point F - the location of point C after the expansion of the circle. If by our definition of distance we depend upon this time (as we do with the current, year 2015, standard definition of the metre, for example), then AF is the distance AC - even though they are geometrically distinct in length. If, however, we somehow "know" the arc distance AC then that arc distance is the distance.

This looks confusing. In fact, we must always make it clear what sort of distance we are talking about - whether we are measuring the distance from A to C with both points at time $a$ or whether we are measuring the distance from A at time $a$ to C at time $a+t$ (point F ), to allow for the measurement signal to pass or by some other measure. Note that A and C are not moving with respect to each other, and $G$ and $F$ are not moving with respect to each other. $F$ is moving with respect to C only in the time (radius, T ) direction, and also G is moving with respect to A only in the time (radius, T ) direction.


Figure 5
Often we measure distances by reflecting light from the source point A to the target point C and reobserving at A - but in this thought-experiment that point A no longer exists, but we have as in Figure 5, a new point L. More time has elapsed, and in this model some measurements of distance depend upon time.

Now, with the light-reflection, the distance AC may be measured as being (i.e. mapped on to) the mean of the lengths $\mathrm{AF}+\mathrm{FL}((f+l) / 2)$ in units of Ds , or as being the mean of the lengths AG+GL $((t+s) / 2)$ in units of T.

Depending how we define the constancy of light speed, we may or may not have that $t=s$. For the next part of this discussion we will assume that we do not have this equality - namely $t \neq s$, and (in general) $s>t$.

In general, $f=a \tan (\theta)$ and $l=(a+t) \tan (\theta)$, so that the mean length (in the Ds units) is

$$
\begin{aligned}
& (2 a+t) \tan (\theta) / 2 \text {. We also know that } a+t=a \sqrt{1+\tan ^{2}(\theta)} \text { and also } \\
& a+t+s=\sqrt{a^{2}\left(1+2 \tan ^{2}(\theta)\right)+t \tan ^{2}(\theta)(2 a+t)}=\sqrt{a^{2}+\tan ^{2}(\theta)\left(2 a^{2}+2 a t+t^{2}\right)} .
\end{aligned}
$$

For very small angles $\theta$ this is very nearly Euclidean - and in local space we normally take it to be Euclidean. But for larger angles - that is, greater distances - we know that space is not quite Euclidean. If this particular model is useful it has some consequences:

- [under some definitions] The speed of light is changing over time, hence
- Using light to measure distances requires a computational correction.

Note that the change in the speed of light depends on the way we define both distance and time. If both of these are defined in terms of the speed of light, then (under those definitions) light-speed does not change. Currently (March 2015) distance is, within the definition of the metre, defined according to light-speed ("the length of the path travelled by light in vacuum during a time interval of $1 / 299,792,458$ of a second"). The second (unit of time) is currently defined as "the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom" which may well itself be directly related to light-speed. Thus under current definitions of the metre and the second, light-speed may be invariant - but it is so by definition.

If we adopt a different definition of the metre, which is not related to light-speed (though this may be very difficult, in the abstract, to do), and a different definition of the second which also is unrelated to light-speed, then with these new unit definitions we may find that light-speed varies. These unit definitions, within this thought-experiment, would have to have the same magnitude as the BIPM definitions "locally". That is, their magnitudes would have to match the BIPM magnitudes within the Solar System region of space, and within the local (current) time period. It may, of course, turn out to be impossible to make a coherent definition of either spatial or temporal separations without reference to light-speed - it may be impossible, in the real world, to make the alternative unit declarations.

## Measuring Time

Just as there is some possible ambiguity about how (spatial) distance can be expressed, there is a similar difficulty with time.
If the rate of expansion of the circle is constant, then there is no difficulty: there is a single time dimension which may be measured as being the difference in the circle's radius at each of the two events (for events stationary with respect to the observer), or the perceived differences in radii by the more general observer. In this case, there is only one time dimension. In this case, Hubble's Constant is actually constant.
Bot if the rate of expansion is not uniform across time, but varies, then there are two different measures of time. One measure is exactly as it was in the uniform-expansion case, but the other independent of it because of the non-regularity - is measured "from the first origin". If we consider all the circles back to the origin all taken together, and for illustration packed up, on on the other, then we get a diagram - a solid - which indicates the rates of expansion. In the case of a constant rate this figure is a cone. In the case of an expansion which tails off we have the solid of rotation of a parabola or ellipse or similar. In the case of an expansion which first increases, then tails off, then increases once more we have a bell shape - and so on, for all the hypothetically possible changes in expansion rate we wish to consider. In all of these non-uniform cases, Hubble's Constant is not constant (see Figure 9).
Whatever the shape of this solid, a cross-section of it is a view for some observer of some spacetime events having some common property. One of the cross-sections for one specified observer is the set of events visible to that observer. Another cross-section (orthogonal to the solid's major axis) is the "now" events, applying a mythical Newtonian universally synchronous clock ("God's clock"). Another might follow a chain of events which are spatially moving with respect to one observer, but moving at a rate which is neither that of the speed of light, nor that of the universal expansion rate and so on.


Figure 6: Conical Space-time (Regular Expansion)


Figure 7: Irregularly Expanding Space-time


Figure 8: Space-time with Time Projections

We are used to the idea of multiple dimension where space is concerned: it is less easy to get an intuitive picture of multiple time dimensions. Only if the rate of expansion is variable - the Hubble Constant is not actually constant - are these multiple time dimensions actually independent of each other. Again, this is a topic, fully open for discussion, that has both theoretical and experimental implications - is the overall Time dimension best expressed as two (or more) independent dimensions? Would it be more convenient to express Time as a complex number? Are there experimental conditions that can show two (or more) independent (and different) times for the same events for the same observer?
In Figure 8 we have an illustration of two different ways of measuring the time between events B and B' from the point of view of observer A. One measure of time is along the line BB', which lies on the surface of the expansion body. Another measure is the distance TT' which is between the orthogonal projection from BB’ onto the axis of the expansion body. If the expansion rate is
constant over this interval then there is a direct relationship between BB' and TT' but if the expansions rate changes over that period then (in general) these two methods of expressing a time interval are independent of each other, but can be mapped using the varying values of the equivalent of the Hubble Constant.


Figure 9: Hubble's "Constant"
(Note that these diagrams do not illustrate the same abstract image of space used elsewhere in this publication.

## What are we Measuring?

We are measuring time, and we are measuring distance. But there are several different definitions of distance, and several different definitions of the time that we should be measuring.

## Distance

In particular, if we consider the expansion picture of the universe, similar to that presented in Figure 8, we have two distinct definitions of the spatial separation between what we may perceive as being two events (but are actually three events). That is, consider the distance between A and B. This is measured along the surface of the expanding space-time diagram. But the distance between A and $B$ ' is measured along a notional line of "now:, without taking expansion into consideration. For very short distances - say, just a few metres - there is negligible difference between $A B$ and $A B$ ', but for larger distances there is an appreciable difference between them.


Figure 10: Alternative Distances
When we state Hubble's Constant we have to be sure that we are always using the same definition of distance for all our measures. And when we quote that constant we need to be clear whether we are talking about expansion along the "now" line ( AB '), or upon the surface $(\mathrm{AB})$ of the space-time diagram.
Time
We are also measuring time. If the definition of the time unit is related to the speed of light, then we have some conceptual difficulties in calculating any revised light-speed - not insurmountable, but points of care that have to be considered. The current SI/BIPM definition of time ${ }^{4}$ does not, explicitly, refer to the speed of light - but we need to be sure that is not so related, or that the extent of any such relationship is known.

[^3]
## Change of Light-Speed.

If there is a change in light-speed, this is not now a large rate of change, and as the universe gets older and continues to expand, the relative change of speed continues to get smaller. Consider the difference between $t$ and $s$ in Figure 5. As an example let us take $a$ to be 1.3E26 metres or 4.35E17 seconds, and take a sample $t$ to be 1E10 seconds, which is about 317 years.
Since $\tan ^{2}(\theta)=t / a$ we have $\tan ^{2}(\theta)=(1 \mathrm{E} 10) /(1.3 \mathrm{E} 26) \approx 1 \mathrm{E}-16$ hence $\tan (\theta) \approx 1 \mathrm{E}-8$. We have $f^{2}=l^{2}-(t+s)^{2}$ and $f /(t+s)=\tan (\pi / 2-\theta)=\cot (\theta)$ hence $f \tan (\theta)=t+s=a \tan ^{2}(\theta)$ and $s=t-a \tan ^{2}(\theta)$. Now given the figures in our example, the difference between $s$ and $t$ is about 4 seconds. That's 4 seconds in 317 years - a significant, but not large, difference.
That would mean, by this rough calculation, that the speed of light is currently slowing at the rate of four parts in ten to the ten. This is a controversial suggestion. According to the BIPM (http://www.bipm.org/en/measurement-units/base-units.html) the definition of the second may be correct to one part in ten to the sixteen - but the observation of the speed of light is accurate to only one part in ten to the ten. This is better than the four parts in ten to the ten being suggested here for the speed of light change.

There is another way in which light-speed may appear to vary. Imagine light transmitted from A to C (eventually arriving at F). At each point on its path the light is travelling at local light-speed. If we imagine a series of initially equally spaced points on the arc AC these map onto a series of unequally spaced points on the line AF - as we go from A to F the divisions become further apart. And though at each point light is still travelling at local light-speed, because the space itself is increasing, the overall speed of the light on the line AT appears to be slowing down. It is not slowing down - it's the space that's becoming bigger.

If a light-signal always travels at the local light-speed, and that speed is always the constant c, we still get the observation that over long distances the "remote" and "local" light-speeds appear in some ways to be different - the remote light-speeds appear to be higher than the local light-speeds, as the time taken to transit certain distances is shorter than can be achieved at speed c - but this does not contradict the edict of constancy of light-speed, as we know about the expansion of space. The "distant" space-intervals, although known to be "now" of a certain magnitude, are seen not "now" but "then" (i.e at the time the light-signal transversed them). We have to be very careful when talking about distances which sort of distances we are discussing - chord distances at the instant (Dc), arc distances at the instant (DA) - both of which can only be calculated, not directly observed - or Ds (tangent) distances which are the ones we observe using light-signals.

In the real world there is universal expansion: but that expansion is not uniform, as it appears to not be taking place inside small systems that are locally bound. We do not appear to see points on earth moving apart from universal expansion, for example, nor points within the Solar System. The magnitude of universal expansion within a small space, however, is very tiny and it may be that this expansion does still take place inside small locally-bound systems, but we (as yet) have not performed any experiments of sufficient sensitivity to detect it - at present this author does not know. Whether it is the case or not does not affect the overall image being considered here, however - over long transmission distances we do have to take account of the apparent change in lightspeed.

## Computational Correction

If the speed of light really is changing by any of these means - and it is not generally accepted that it is - then to use light signals as a means of measuring distance means that we have to correct the calculations by the speed-change amount. For "small" distances (say, less than ten thousand lightyears) this correction can be completely ignored - other errors completely swamp it. But for "large" distances (say, several millions of light-years) then this correction does need to be applied.

There are several scales we need to consider here - my estimate of "small" above may be completely the wrong one. In fact there are (at least) eight levels of scale that we have to consider:
(i) Earth, (ii) Solar System, (iii) Solar Interstellar Neighbourhood, (iv) Milky Way Galaxy, (v) Local Galactic Group, (vi) Virgo Supercluster ("our" supercluster), (vii) Local Superclusters (superclusters other than "ours"), and (viii) Observable Universe.

Each of these scales has a range of theta - a range of distance. For some of these we have to consider the diameter of that region, but for the largest only its radius for observation (and its diameter for other theoretical computations). In the table below distances are in metres in direction Ds, and we assume the current observable radius of the universe (direction T) to be 4.3E26 metres. Computation is from $\theta=\tan ^{-1}(f / a)$ :

| Region | Nearest | Furthest | $\theta$ | Comments |
| :---: | :---: | :---: | :---: | :---: |
| Earth | 0 | 1E8 | < $0^{\circ} 0^{\prime} 1.4 \mathrm{E}-17{ }^{\prime \prime}$ | Not as far as the moon - but including artificial satellites |
| Solar System | 1E2 | 1E15 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.3 \mathrm{E}-21^{\prime \prime} \\ & <0^{\circ} 0^{\prime} 1.4 \mathrm{E}-19^{\prime \prime} \end{aligned}$ | Radius of about $300 \mathrm{AU}^{5}$ |
| Solar Interstellar Neighbourhood | 1E6 | 1E19 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.3 \mathrm{E}-17^{\prime \prime} \\ & <0^{\circ} \\ & 0 \end{aligned}$ | Radius of about 150 parsecs $^{6}$ |
| Milky Way Galaxy | 1E7 | 1.2 E 21 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.3 \mathrm{E}-17 " \\ & <0^{\circ} \\ & 0^{\prime} \\ & \hline 1.6 \mathrm{E}-4 " \end{aligned}$ | Radius of about $40 \mathrm{kpc}^{7}$ |
| Local Galactic Group | 1E15 | 1E23 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.3 \mathrm{E}-10^{\prime \prime} \\ & <0^{\circ} \\ & 0^{\prime} \\ & 1.4 \mathrm{E}-2 " \end{aligned}$ | Radius more than $2.6 \mathrm{Mpc}^{8}$ |
| Virgo Supercluster | 1 E 17 | 6E24 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.4 \mathrm{E}-8 " \\ & <0^{\circ} 0^{\prime} 8.0 \mathrm{E}-1 " \end{aligned}$ | Radius more than 18 Mpc |
| Local Supercluster | 1E18 | 1E25 | $\begin{array}{lll} >0^{\circ} & 0^{\prime} 1.3 \mathrm{E}-7 " \prime \\ <0^{\circ} & 0^{\prime} & 1.4 \mathrm{E}-1 " \end{array}$ | Radius more than 33 Mpc |
| Observable <br> Universe | 1E19 | $\begin{aligned} & \text { 4.3 E26 } \\ & \text { (8.8E26) } \end{aligned}$ | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.4 \mathrm{E}-6^{\prime \prime} \\ & <44^{\circ} 59^{\prime} 59^{\prime \prime} \end{aligned}$ | Quoted up to the observable diameter, though we can see only half this distance (observable radius) |

Because of the scale of the universe, most of these angles are very tiny - it is only when we get to more than 5E24 metres that the angles (and hence the measurement errors) are appreciable.

[^4]
## Observation at a Distance

Consider what an observer at point C can see of point A. Information coming by light from point A arrives when the observer is at point $C^{\prime}$. Thus the radius of curvature at A seen from C' is different from (and smaller than) the radius of curvature at C' seen from C'. And as we consider A and C being further and further apart, the greater that difference in perceived curvature (see Figure 11).


Figure 11
And this is true for all observers - wherever you are, as you look out on the universe, the further away you look, the more curved that part of space appears to be. So far we have been considering an observer looking outwards - but in fact we are mostly looking back in time... we are seeing distant objects not as they are but as they were in the past. Hence we have another set of correspondences between angles, times and distances. These are for an observer at point C (becomes C’) looking at point A. The Time column indicates the light-speed transfer for the Nearest and Furthest distances, and is measured in seconds. Remember that the universe is currently considered to be just 4.35E17 seconds old, and no observed light-time distance can exceed that.

| Region | Nearest | Furthest | $\theta$ | Time |
| :---: | :---: | :---: | :---: | :---: |
| Earth | 0 | 1E8 | < $0^{\circ} 0^{\prime} 1.4 \mathrm{E}-17{ }^{\prime \prime}$ | $\begin{aligned} & 0 \\ & 3.335 \mathrm{E}-1 \end{aligned}$ |
| Solar System | 1E2 | 1E15 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.3 \mathrm{E}-21^{\prime \prime} \\ & <0^{\circ} \\ & 0 \end{aligned} 0^{\prime} 1.4 \mathrm{E}-19^{\prime \prime}$ | $\begin{aligned} & 3.335 \mathrm{E}-7 \\ & 3.335 \mathrm{E} 6 \end{aligned}$ |
| Solar Interstellar Neighbourhood | 1E6 | 1 E 19 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.3 \mathrm{E}-17{ }^{\prime \prime} \\ & <0^{\circ} 0^{\prime} 1.4 \mathrm{E}-15^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 3.335 \mathrm{E}-3 \\ & 3.335 \mathrm{E} 10 \end{aligned}$ |
| Milky Way Galaxy | 1E7 | 1.2E21 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.3 \mathrm{E}-17^{\prime \prime} \\ & <0^{\circ} 00^{\prime} 1.6 \mathrm{E}-4 " \end{aligned}$ | $\begin{aligned} & 3.335 \mathrm{E}-3 \\ & 4.002 \mathrm{E} 12 \end{aligned}$ |
| Local Galactic Group | 1E15 | 1 E 23 | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.3 \mathrm{E}-10^{\prime \prime} \\ & <0^{\circ} 0^{\prime} 1.4 \mathrm{E}-2 " \end{aligned}$ | $\begin{aligned} & \text { 3.335E6 } \\ & \text { 3.335E14 } \end{aligned}$ |
| Virgo Supercluster | 1 E 17 | 6E24 | $\begin{array}{lll} >0^{\circ} & 0^{\prime} 1.4 \mathrm{E}-8 " \\ <0^{\circ} & 0^{\prime} \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 3.335 \mathrm{E} 8 \\ 2.001 \mathrm{E} 16 \end{array}$ |

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| Local Supercluster | 1 E 18 | 1E25 | $\begin{array}{llll} >0^{\circ} & 0^{\prime} & 1.3 \mathrm{E}-7 " \prime \\ <0^{\circ} & 0^{\prime} & 1.4 \mathrm{E}-1 " \end{array}$ | $\begin{array}{\|l\|} \hline \text { 3.335E9 } \\ \text { 3.335E16 } \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| Observable Universe | 1E19 | 4.3 E 26 (8.8E26) | $\begin{aligned} & >0^{\circ} 0^{\prime} 1.4 \mathrm{E}-6 " \\ & <44^{\circ} 59^{\prime} 59^{\prime \prime} \end{aligned}$ | 3.335E10 <br> 1.43E18 (longer than the current age of the universe) |

In fact, the relationship between distance and time is not completely simple: other factors come into play, and the red shift is one of them. The relationship between distance and redshift ( $z$ ) is illustrated in Figure 12 below.

The two distances in this diagram $\mathrm{d}_{\mathrm{H}}$ and $\mathrm{ct}_{\mathrm{LB}}$ are respectively the Hubble distance (comoving distance with respect to Earth), and the light-time distance. The light-time distance is limited above to the age of the universe: the Hubble distance, however, is not.

There are a number of possible means of stating distance already in use, in conventional physics. All of these are close in value at shall distances, but diverge at larger distances. Some of these are indicated in Figure 13. The one we shall be using here is the rightmost of these - which is according to the Lambda-CDM model. The four different types of distances graphed are DL, the luminosity distance; Dnow (the same as dH in Figure 12), the Hubble distance; Dltt (the same as $\mathrm{ct}_{\mathrm{Lb}}$ in Figure 6), the light time travel distance; and Da the angular size distance (not the same as the Da used elsewhere in this document).


Figure 12

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Figure $\mathbf{1 3}^{9}$
For a partial, and very beautiful, illustration see [4].

[^5]
## Base Question Reconsidered

So - What is the distance between two points? And also - linked with this - What is the time between two events?

We already know, from Relativity, that there is no absolute distance between two events - the distance depends upon the observer, and how that observer is moving relative to the events being measured. Similarly "What is the time between two events?" also depends upon the relative movement of the observer with respect to the events.

There is also, within Relativity, the concept of curvature of space. That is not the same as the curvature previously mentioned here, but another local curvature superimposed upon the expanding circle as we have described it. The Relativity curvature (associated with, amongst other things, the presence of gravitational objects) may be in the same plane as the circle (which would effect the measurement of absolute time for each observer), or in a direction orthogonal to it (which, by itself, would not - for absolute time considered as a direct projection from the OA radius).

Let us now remove from this thought-experiment the concept of observable absolute time. If we take the Relativity space-time curvature to be at right-angles to the OA (absolute time) radius, then that curvature does not effect the absolute time (the projection of the OA radius onto the plane of expansion of the circular arc). But if we consider time, for each observer, to be measured along the OA line itself ${ }^{10}$, as the point/event A may be at various heights (because of Relativity curvature) above that plane, then time as measured by each observer depends upon that observer, and the space-time curvature for that observer. No observer can know, in any way, the absolute curvature at his part of the universe: each observer sees the universe in his small locality as being flat and Euclidean. Each observer sees a curvature at other parts of the universe. Of course, for convenience in calculation, we often consider the relative curvature of the universe at two different events, and allow for local curvature at the observer's location - but that is not absolute curvature (which is unobservable) but relative curvature (relative between two observed events).

This also removes the concept of observable absolute distance. If the absolute distance is the projection onto the plane of any of the distance measures so far considered (arc AB , chord AB , line AF, etc.) then it takes no account of Relativity curvature. If, however, distance as measured is always in terms of the actual locations (in the thought-experiment space), where events may be above that plane by large or small amounts, then that observable distance too depends upon the relative curvatures for the two events.
(These absolute/observable projections may, in this thought-experiment, be the wrong way round: it may be that the observable is what is in the plane, and the absolute is what is above the plane - that is something to be considered in another publication.)

In a future paper I shall consider the mathematics behind this thought experiment, and also look into the precision limits of the measurement of the velocity of light.

[^6]
## References

[1] See http://en.wikipedia.org/wiki/Metric expansion of space (examined 2015-04-04) for an extensive list of references
[2] See http://en.wikipedia.org/wiki/Distance (examined 2014-11-27) for some description, and for a further list of references
[3] See also http://en.wikipedia.org/wiki/Comoving distance (examined 2014-11-27) for a description of some methods of describing distance measures, with further references
[4] http://en.wikipedia.org/wiki/Observable universe (date 20141119) ("Observable universe logarithmic illustration" by Unmismoobjetivo - Own work. Licensed under Creative Commons Attribu-tion-Share Alike 3.0 via Wikimedia Commons -
http://commons.wikimedia.org/wiki/File:Observable universe logarithmic illustration.png\#mediav iewer/File:Observable universe logarithmic illustration.png)
[5] http://www.astro.ucla.edu/~wright/cosmo 02.htm\#DH examined 2014-11-19
[6] http://en.wikipedia.org/wiki/SI base unit inspected 2015-04-02, which refers to International Bureau of Weights and Measures (2006), The International System of Units (SI) (8th ed.), ISBN 92-822-2213-6
[7] http://physics.nist.gov/cuu/Units/second.html inspected 2015-04-02, referring to the 1967 and 1997 definitions

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[^0]:    1 Contactable at idkk@idkk.com All the errors are my own.

[^1]:    2 There are numerous definitions of distance used within physics [2], and each definition gives a different quality to the measure. We may be talking of Euclidean distance, Minkowski distance, directed distance, comoving distance (Hubble distance), light-time distance or proper distance [3] - each of which has a precise (and sometimes nonintuitive) definition.

[^2]:    3 The E in the notation for very large and very small numbers indicates the power of ten. Thus one million is 1E6 and one millionth is $1 \mathrm{E}-6$, and so on.

[^3]:    4"The second is the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom" and "This definition refers to a caesium atom at rest at a temperature of 0 K ". [6][7]

[^4]:    5 Astronomical Unit - about 1.5E12 metres
    6 Parsec - about 3.08E16 metres
    7 kpc = kiloparsec - about 3E19 metres
    $8 \mathrm{Mpc}=$ megaparsec - about 3 E 22 metres

[^5]:    9 Figure 12 and Figure 13 are from [5]

[^6]:    10 Or by some other, possibly multiple, means - see the section on Measuring Time, page 8 above.

