Dimensions of the Universe – II

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Fundamental Questions – II

In a previous document (*Dimensions of the Universe-I*) [1] we considered two fundamental questions which we pose in many circumstances:

- What is the distance between two points?
- What is the time between two events?

We already know that these questions are improperly put: from the analyses which have given rise to Relativity Theory we know that a better form of these two questions, which must be taken together, is:

- What is the space-time distance between two events?

In our ordinary experience – and by "ordinary" I mean here as we sit in our homes, or wander round the streets, or build sheds in the garden – we think of the world as having three, orthogonal directions which measure out a Euclidean space, plus one independent Time direction. And – in our ordinary experience – all of these measures and sets of coordinates are shared by everyone.

We do not have to move far out of our kitchens or away from our back yards to realise that this ordinary viewpoint – the view shared by most seven year olds in the world – is flawed. We can observe that in free space light does *not* always travel in straight lines; that we are on the surface of a sphere and not a flat plane; that time moves at different rates for different observers (and to get GPS to work we have to take account of this [11]); that Newton's beautiful gravitational formula of $G m_1 m_2 / r^2$ does not *quite* work when calculating the orbit of planet Mercury [12]; and that the further away things are, the faster they are receding from us [13][14] – and that that is true for all observers. We now know that one set of coordinates – both space and time measures – does not work for all observers. Worse than that – one set of coordinates works for two different observers only in a very restricted set of cases: strictly speaking, there is a separate coordinate system for every observer.

We have formulae that help us convert from any one system of coordinates to any other – though sometimes these conversions are complicated, and contain variables (physical constants) of whose values we are uncertain to some degree. The conversion formulae variously depend upon G (the gravitational constant), c (the speed of light in vacuum), h (Planck's Constant), e (the elementary charge); m_e (the electron mass), m_p (the proton mass), α (the fine-structure constant), m_u (the atomic mass constant), k (the Boltzmann constant), F (the Faraday constant) – and so on, for many more.

These formulae work very well: but they are only as accurate as the measurements of the fundamental physical constants on which they depend. If, for example, we know G with an uncertainty of one part in 10^4, then all formulae containing G are affected in the precision of their accuracy too – and likewise for each of the other physical constants.

We can define units of measure (for example, Planck's Units), which within themselves subsume some of these physical constants, and are by definition independent of any human construct – but we still have to produce human-made machines to measure real events against these units of measure, and these (again) can be only as accurate as our knowledge of those fundamental constants. With Planck's Units we are dependant upon c, G, $\hbar$ (the Reduced Planck Constant), ε_0 (the electric

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constant), and k_B (the Boltzmann Constant) – and hence upon our knowledge of their magnitudes.

The Speed of Light

At present (the year 2015) in the BIPM definition of the metre length and the second of time we include the speed of light in vacuum, *c*, as a known number which we take as absolutely known, and upon which our units of measure depend. Thus we cannot (using the current systems of measure) simply ask "Is the speed of light in vacuum equal to 299792458 metres per second, or to some other value?" as that very number is part of the definition of the metre itself. Despite that, it is still a meaningful question to ask "What is the speed of light in vacuum?", but we have to be careful as to how we define the units of measure that we are using and against which we phrase the answer.

In what follows I shall be using the 2015 (BIPM) definition of the metre [2], this is *not* assuming that we have an instantiation of the metre measure (for example, two scratches upon a platinum-iridium bar in Paris) which match that definition precisely. Unless otherwise specified, it is always to the BIPM metre that I am referring: the marked bar is *not* the metre.

In order to determine a speed we usually measure the time of movement between two points of known spatial separation (known distance). We then divide distance by time to get $s = v/t$ and express the result in, for example, metres per second. Our speed measurement, if performed in this simplistic way, is affected by the precision with which we can (a) measure the time of movement, and (b) measure the spatial separation (distance). As technology improves our precision of measurement improves.

In 1975 the speed of light was, by measurement believed to be of the order of 299792456.2 ± 1.1 metres per second – to within 3.5 parts per thousand million [15]. This is a known measurement uncertainty. Other observations gave a more precise measurement. Subsequent to 1983, with the change in definition of the metre in terms of the speed of light, measurements of the speed of light actually serve to better define the length of the metre.

To better know the length of the metre (or the speed of light) we have to make measurements to better accuracy than 3.5 per thousand million[2] (3.5E9) or, under some descriptions, two parts per hundred thousand million (2E11). Can we do this?

Theoretical Limits of Precision and Accuracy

Thought and Real

Precision is the grouping together of the measured results; Accuracy is their relationship to the real world value being measured.

In the thought-experiment universe we can make completely precise and accurate measures. In the real universe, however, we are constrained by (amongst other things) moving reference frames, quantum uncertainty, observation error, whether relativistic effects are constant over the time of measure, the mapping from real events to abstract points in a theoretical space-time space, and so on.

There are many methods available for measuring light speed – some of these use time and distance measurements, others deduce light speed from measuring frequency and wavelength, or from vacuum permittivity ε_0 and vacuum permeability μ_0. I would like to consider here three methods of measurement – two of which have actually been used, and one which is only a thought-experiment. The three methods differ greatly in the magnitude of the distance used. The lengths used in these

2 See the section on Representation of Numbers (page 11) below for a clarification of the E notation used here.

three methods are: several hundreds of light years; Earth-based distances (up to a few hundreds of kilometres); and some atomic scale distances. For each of these methods I would like to consider the precision and accuracy or measurement of the times and distances involved.

Astronomical Distances

Hundreds of light years – I am restricting considerations here to astronomical distances of under 500 parsecs[3] (now taken to be 500*3.08567758E16 metres [6] = 1.542838790E18 metres – for sake of argument, let us take our limit to be 1.5E18 metres). For astronomical distances greater than this the errors of estimation are very large – at best we think we are within 5% of the true distance, with accuracy continuing to reduce as the distance grows, to possible errors of well beyond 50% of the true value. Such imprecise distances cannot give us a useful base for measuring light speed with either precision or accuracy.

For astrophysical purposes we need to know the speed of light – but despite the various measures used for astronomical distance measurement, we cannot get from astronomical observations alone an accurate estimate of light-speed.

Terrestrial Distances

For experiments done on (or near) the Earth we can measure distances very accurately. By using lasers to measure distance, and atomic clocks to measure time we can get up to an accuracy of 1 part in 1E16, though this is not normally achieved. The most precise light-speed value known in 1973 was 2.997924574E8 ± 0.1 metres per second. This is known to be accurate to 5 parts in 1E10 Subsequent measurements have brought the uncertainty down to 2 parts in 1E11.

Terrestrial experiments might include using the Earth-Moon distance as base-line, which gives us an accurately-known, but varying, distance of about 3.84E10 metres known to a precision of better than 3.8 cm (3.8E-2 metres). This is a precision of slightly better than 1 part in 1E10. Unfortunately this long base-line “wobbles”, as the Moon moves towards and away from the Earth.

Atomic clocks allow us to measure time intervals to a precision of one part in 1E14 – that is 1E-9 seconds per day. The declared precision of the clock NIST-F2 is 2.3E16 [16] – an even better precision, but possibly not yet applicable to real measurement use. For the purposes of this document, however, we shall assume that we can measure to one part in 1E16 – even better than has yet been achieved.

With these two precisions the resultant speed can be known only to the lesser precision of one part in 1E10. But other techniques – not reliant upon visible distances, but very short distances, and the measurement of frequency have allowed us to gain the extra order of magnitude in precision. We can construct waves (usually of laser light) of a known frequency, f and measure its wavelength λ with very great precision. Then the formula $c=\lambda\times f$ gives us the speed of light c. If visible light is used as the measuring rod, then the wavelengths are between 4E-7 and 7E-7 metres. If ultra-violet light is used, then the wavelength can be as little as 1E-8 metres, on the border of the X-ray band. If we take colour of light used to be of wavelength 8E-7 metres (in the near ultra-violet), then the best precision that can be guaranteed is to measure a length to within 4E-7 metres

3 A parsec is about 3.261 light-years. It is the distance of an object that subtends one arc-second of parallax at the distance of 1 AU. For observational purposes, it is the distance of an object that subtends two arc-seconds of parallax for observations made 2 AU apart – at six month intervals, with the Earth on opposite sides of the Sun. Since the AU is now defined as being 1.49597870700E10 metres [4] – even though, because of the ellipticity of the Earth’s orbit it in fact deviates from this – the parsec can now be defined as 3.0856776E16 metres [6]. This is only as accurate as (a) our knowledge of the actual value of the Earth-Sun distance at the times of observation, and (b) the precision and accuracy of measuring the very small angles involved.

(half wavelength). If we then take the major base-line to be 8 km. (there and back inside a vacuum tube 4km in length) then we have a length measurement precise to 4E-7 meters in 8E3 meters, which is a precision of 5 in 1E12.

The above description is an over-simplification, but gives some idea of the precision (and, we hope, accuracy) already achieved. By projecting these figures forward in time, and assuming that are measurements will become even more precise, we shall say within this document that our knowledge of the speed of light in vacuum is accurate to one part in 1E16 – which is more precise than we currently (year 2015) actually know it.

Atomic Distances

Consider a very short baseline – one of atomic length. We shall consider two such lengths here.

We take the classical radius of an electron as our first example. This length does not represent anything particularly significant, but is of the atomic scale. It is about 2.8179403267(27)E-15 metres [17], where the parenthesised numbers are a short way of representing the uncertainty in the preceding digits. In this case that means 2.8179403267E-15 ± 2.7E-24 metres for the classical electron radius.

One Planck Unit of Length l_P is 1.616199(97)E-35 metres [2][6]. Hence the classical electron radius is about 1.74356024641768E20 l_P. There is a suggestion – a very strong suggestion – that we cannot measure any distance, even in the abstract, to an accuracy of smaller than one Planck Length. This would mean that we are constrained to measuring the length of the base-line we are considering here to no better than about 1.7435 parts in E20 – still very accurate, but an upper limit.

From the definition of the Planck Units we know that the time taken for light to transverse this distance is also 1.74356024641768E20 t_P where one t_P (the Planck Time) is 5.39106(32)E-44 seconds [2][6]. Again, we cannot measure times to an accuracy of smaller than one Planck Time. This gives us another limit of about 1.7435 parts in E20.

These theoretical accuracy limits, taken together, limit the usefulness of our measurements over this very short base-line to about 3 parts in E10 – not as good as we believe we have already achieved (and, in the future, will achieve) over longer base-lines. In this first calculation I have not taken account of our uncertainly of the base figures themselves – that contributes errors many magnitudes smaller, and can, in this first thinking, be ignored.

If we take another, even shorter, base-line which does represent something significant – the diameter of a neutrino – then we have another set of error limits.

As with all atomic particles, to speak of a radius or a diameter is to simplify what is a much more complex real situation. An electron, for example, does not have the same sort of size as, say, a billiard ball: a billiard ball is a simple sphere, with a specifiable diameter and finite and determinable surface area. An electron, though, may be mapped for some purposes of calculation to a finite particle – but for other purposes we have to take account of its wave, dispersed structure. An electron is not anywhere specific: it is everywhere. Of course, there is a centre of probability for that electron’s effect, and we can think of the electron as being “there”, with its effects fading rapidly away in all directions. But we have to remember that these effects are never zero.

And I mean *never*. An electron “here” in the tea in the cup on my desk, has an effect upon the rotation of the Andromeda galaxy. Not a big effect, I agree – in fact it is an utterly minute effect which, individual electron by individual electron, can usually be ignored. But it is also clear that the totality of the Earth – its whole mass – has an effect upon Andromeda (a very small one), and the mass of the whole Solar System likewise (larger, but still very small). For most calculations of Andromeda-distance effects, though, we ignore the individual electrons, or the planetary-sized particles, or the Solar System sized "particles" and stick to the mass of the whole Milky Way galaxy considered as a whole, or at most separated out into large structures like "arms", which themselves contain millions of stars and sub-systems.

Just as we have a classical radius for the electron (mapping it to a contained particle like a billiard ball – even though we now know that this is a limited and incorrect image), we have a cross-sectional area for a neutrino. Depending on the type of neutrino, this area is of the order of 1.0 to 3.2E-37 m^2, giving a notional diameter of about 2E-19 m.[5] This is very much smaller than the electron. If this length is expressed in Planck Units it is of the order of 1.24E16 l_P. Again, by the definition of the Planck Units, light takes 1.24E16 t_P to pass this distance. Combining the two inevitable errors of one part in 1.24E16 gives us about one part in 1.53E8 for errors in measuring light-speed over this very short distance.

Vacuum

What is Vacuum?

"What is Vacuum?" seems to be a simple question to answer: Vacuum is the absence of all matter within the volume under consideration.

But, alas, there is a difficulty here. Since all atomic particles are wave functions, then there is *no* part of the universe that is completely free from the influence of these waves. Granted, we can find very large volumes of intergalactic space that contain even less matter than one atomic particle per cubic metre – but there are always radiation effects.

Refractive Index

The Refractive Index of a perfect vacuum is, by definition, 1. That is, light travels through it at the speed of light (which sounds like a truism). If the refractive index of any medium is n then light travels through that medium at velocity v where $v=c/n$.

Since even intergalactic space is not complete empty – is not a completely perfect vacuum as in the abstract definition – its actual refractive index is not exactly 1, but a number very close to 1. For most purposes we can ignore this error, because it it so small. Let the refractive index of free, intergalactic space be $(1+\epsilon)$ where ϵ is a very small number.

How small?

This is not an easy question to answer, and in considering it I am at first going to take space (interplanetary, interstellar, intergalactic and void) as being filled with gas – admittedly a very tenuous gas. I am then going to see whether we can calculate refractive index from pressure and temperature. But even if we can get an order-of-magnitude figure for this, there are other factors influencing refractive index which are beyond the considerations of this publication. For a complete theory, these factors *will* have to be considered.

Pressure

For an ideal gas (n – 1) is proportional to the density of the gas. This means it is proportional to the pressure, and inversely proportional to the temperature. The minimum pressure of intergalactic space is not zero. What that minimum pressure actually is, I do not know. For the purposes of this document I am going to assume that it is the pressure given by the equivalent of one hydrogen molecule per cubic metre: this may be too much, but may be used to illustrate the calculations. Similarly, the temperature of intergalactic space is not zero.

Gas pressure for an ideal gas can be calculated from the equation $PV=aRT$ where *P* is pressure, *V* is volume, *a* is the amount of gas, measured in mols, *R* is the ideal gas constant, which is itself the product of the Boltzmann Constant and the Avogadro Constant and *T* is the temperature in degrees Kelvin. Pressure, here, is measured in Pascals, Pa. The gas in space is far from ideal (certainly not over the complete range of temperatures) – but this equation would give a first approximation. We will use a different equation later to cater for the non-ideal nature of interstellar gas.

In one cubic metre the volume *V* is 1 m^3, *R* is about 8.314 $J\ K^{-1}\ mol^{-1}$ and *a* is (for one hydrogen molecule) the reciprocal of Avogadro's number 1.6605E-24. Since R is itself the product of the Boltzmann Constant and the Avogadro Constant, these cancel out (for this density only), to give us $P=k_B\times T$. The value of k_B is about 1.380E-23 $J\ K^{-1}$. So for an ideal gas:

$$P=1.380\text{E-}23\times T \qquad (1)$$

For a non-deal gas, the Redlich-Kwong equation [7][8] gives a relationship between Temperature *T*, Pressure *P*, and Molar Volume V_m:

$$RT=(P(V_m-b))+\frac{a}{V_m(V_m+b)T^{\frac{1}{2}}}(V_m-b) \qquad (2)$$

where *R* is the ideal gas constant (about 8.3144621(75) J /mol /K), *T* is the temperature (in degrees Kelvin), *P* is the pressure (in Pascals), V_m is the molar volume (in cubic metres – note that this is normally quoted in cubic decimetres, hence this figure will be 1E6 times the normally quoted values), and *a* and *b* are constants evaluated by:

$$a=0.4275R^2\frac{T_c^{2.5}}{P_c} \quad \text{and} \quad b=0.0867\frac{R^2T_c}{P_c} \qquad (3)$$

where T_c is the critical temperature and P_c the critical pressure for that gas. For monatomic hydrogen and helium the critical pressures and temperatures are:

	T_c	P_c
H	33.2	1.3E6
He	5.19	2.27E5

The molar volume of hydrogen at the density of one atom per cubic metre is the same in cubic metres as the number of atoms (molecule) in one mol, Avogadro's constant = 6.0221E23. Hence the pressure is given by:

$$P=(\frac{RT}{V_m-b})-(\frac{a}{V_m(V_m+b)T^{\frac{1}{2}}}) \qquad (4)$$

$$\text{or} \quad P=(\frac{RT}{N_A-b})-(\frac{a}{N_A(N_A+b)T^{\frac{1}{2}}}) \qquad (5)$$

In equation (5) we interpret N_A as having the units m^3 and not the standard mol^{-1} .

To a first approximation, taking *R* to have the value 8.3114 then *a* and *b* have the values:

$$a = 0.4275\, R^2 \frac{T_c^{2.5}}{P_c} \qquad \text{and} \qquad b = 0.0867 \frac{R^2 T_c}{P_c}$$

$$a = 0.4275(8.3114)^2(33.2)^{2.5}/1.3\text{E}6 \qquad b = 0.0867 \times (8.3114)^2 \times 33.2/1.3\text{E}6$$

$$a = 0.4275 \times 69.0794 \times 6351.0453 \times (7.692\text{E-}7) \qquad b = 0.0867 \times 69.0794 \times 33.2 \times (7.692\text{E-}7)$$

$$a = 1.4427\text{E-}1 \qquad b = 1.52948\text{E-}4$$

Because of the enormity of N_A we can safely ignore the second term[4] in (5) and give:

$$P = \frac{RT}{(N_A - b)} = \frac{8.3114\, T}{(6.0221\text{E}23) - (1.52948\text{E-}4)} = 1.3802\text{E-}23\, T \qquad (6)$$

For small values of *T* this is a very low pressure.

Better equations than the Redlich-Kwong will certainly exist for the very low temperatures and pressures and ionisation states – but these here are first-approximation figures only

Temperature

The temperature of space is not uniform. Some parts of it are at millions of degrees Kelvin, and others at little above the temperature due to the Cosmic Microwave Background (CMB) [9]. For our considerations here, I am going to assume that "most" of space has the lower temperature of 2.7° K – not much above absolute zero. From all of the assumptions made here (and they *are* assumptions: these calculations will have to be performed again, inserting more figures from the more complex real-world environment), and from equation (6) above, that space pressure is about 3.7265E-23 Pa. In particular, the prior assumptions made here about the temperature of "most" of space is likely to be very wrong.

Interstellar Refractive Index

We know that the difference between the refractive index and 1 is inversely proportional to the gas pressure. Take the constant of proportionality for a given gas *G* to be W_G. Then $(n-1) = W_G \times P$. According to Rouff and Ghandehari [3] the refractive index of molecular hydrogen H_2 for one particular wavelength of light is given by:

$$n_{H_2} = 1 + 3.046\rho + 2.6\rho^2$$

where ρ has units of mol cm^{-3}. If we change ρ to have instead units of mol m^{-3}, bringing us into standard SI units, then the equation becomes:

$$n_{H_2} = 1 + (3.046\text{E}6 \times \rho) + (2.6\text{E}12 \times \rho^2)$$

This is not the same as the linear proportionality first suggested – but for the extremely small values for mol m^{-3} for the very tenuous interstellar gas, we can, for the moment, ignore the term in ρ^2 and rewrite this as:

$$n_{H_2} \approx 1 + (3.046\text{E}6 \times \rho)$$

This formula will have to be reconsidered for monatomic hydrogen ^1H and expanded for both the extreme temperatures of space and for the full range of light (and other) electromagnetic frequencies – there are more relevant pressures in space than just gas pressures. It will also have to be reconsidered according to the Equation of State used to relate ρ (density) to *P* (Pressure), as we may be using the Ideal Gas Law, or van der Waals, or Redlich-Kwong, or Berthelot, or Clausius, or

4 Provided we do not need more than 23 places of decimals.

… insert long list here – but we do have to have a formula that works with (a) extremely tenuous gases, and (b) varying states of ionisation, and (c) mixtures of different gases. Above I have used Redlich-Kwong – but that is not a choice of which formula is correct, merely an arbitrary choice to show some example figures.

With the example values under consideration we have approximately $\rho = 1.6606\text{E-27}$ kg/m^3 which gives the refractive index of space to be very, very approximately $1 + 3.046\text{E6} \times 1.6606\text{E-27}$ or $1 + \epsilon$ where $\epsilon = 5.058\text{E-21}$. (In [18] an even lower value is suggested, at a lower gas pressure than we have taken here, of 4.9354E-23: in this publication we shall continue to use the higher value as our example – the figures will be different, but the overall argument is unchanged.).

If we take the SI/BIPM speed of light in perfect vacuum to be *c* (Einstein's limit speed), then the actual speed of real light in real interstellar space, *C*, is $C = \frac{c}{(1+5.508\text{E-21})}$ which is a difference past the 20th decimal place – beyond the most precise measure we have so far made.

Light Speed and Photon Mass

We have suggested above that:

$$C = \frac{c}{(1+5.508\text{E-21})} \qquad (7)$$

where *C* is the real speed of light in real interstellar space. This is a smaller number than *c* – not by a lot, it is true, but smaller. The Lorentz equation relating rest mass and moving mass is:

$$m_v = \frac{m_0}{\sqrt{1 - \frac{C^2}{c^2}}}$$

or (with these suggested figures, in this particular case) $m_0 = m_v \sqrt{1 - \frac{1}{(1+5.508\text{E-21})^2}}$. This is very roughly $m_0 = m_v \sqrt{(1.1016\text{E-20})}$ or $m_0 = m_v \times 3.319\text{E-39}$.

Because *c* is so large, and the refractive index of "empty" space is so small (*i.e.* close to 1), and m_v for a photon is so small then m_0 for a photon is very small indeed. The theoretical convention at the moment is that m_0 is zero – that is, photons have no rest mass. This is meaningful and experimentally correct, with the current SI/BIPM definition of the metre, only if ϵ (above) is zero. It is, however, my suggestion here that photons have a non-zero rest mass. And I am fully aware that this is an heretical suggestion.

Photon Rest Mass

If photons do actually have rest mass, and interstellar space has a refractive index detectably different from one, then we have some experimental possibilities (some of them not yet realisable – but possibly realisable in the future), and also some theoretical possibilities:

- Can the actual refractive indexes of interplanetary, interstellar and inter-galactic space be (a) calculated and (b) observed? It must be remembered that these are very close to 1, and we are looking at the 20th decimal place and beyond.

- The rest mass of a photon, if it exists, is already known to be less than 1.07E-27 atomic mass units, or 10E-18eV/c^2 or (very approximately) $1.07\text{E-}27 \times 1.6605\text{E-}27 = 1.7762\text{E-}54$ kg. Can experiments be devised that can detect such very small masses for slowly-moving photons in “close” space? (By “close” here I mean “within 100 parsecs of Earth” – though even closer would be better – and by “slowly-moving” I mean at under 0.01*c*).
- Coulomb’s Law does not currently allow the presence of an electric field inside a hollow conductor subjected to an external electric field. This has already been tested to a high precision. The non-zero magnitude of such an interior field resulting from these suggestions could be calculated, and then tested. Can the existence of such very small electric fields be tested?
- We already know that photons inside superconductors do develop a non-zero rest mass. This affects the range of electro-magnetic forces. The non-superconductor effects could also be theoretically calculated and considered. Could such effects partially explain the observed values for universal expansion?
- Are there non-superconductor environments ,which we can create and observe, in which photons come to rest? Are any of the already known sub-atomic particles in fact instantiations of photons at rest, or at very low speeds?
- There are a lot of photons in the universe. Many of these photons are in inter-galactic space, where the gas density, and hence the refractive index, is even lower. Hence these photons are travelling closer to the theoretical *c* (that is, *C* is even closer to *c* than is observed in interstellar space), and hence the moving mass (m_v) of these photons is greater than we normally observe “close” to home. Could this also partially explain the observed values for universal expansion, as each part of the physical universe is “pulled outwards” by the distant photons?
- Can we estimate, either by theoretical or experimental means, the actual number of photons in the universe? Can we describe their global distribution? What are our limits on making precise estimations of these figures?
- In the thought-experiment of an otherwise empty universe containing just one photon, what would be its mass and speed? Would this hypothetical universe be at all similar to the conditions existing within one Planck time of the Big Bang? (This is a particularly conjectural hypothesis).

Representation of Numbers

I am rather shortsighted. Hence I sometimes find it difficult to see the small subscripts which represent the powers of ten used in expressing very large or very small numbers. There is another notation, therefore, used in this document, which uses the upper-case letter E as representing "ten to the power of". Thus I write 10^6 (one million) as 1.0E6. And I write the number 10^{-6} (one millionth), as 1.0E-6. The speed of light is thus 2.99792457E8 metres per second. As examples, the Reduced Planck Constant $\hbar$ is about 1.054E-34 J s, and the Coulomb Constant is about 8.987E9 kg m^3 s^{-2} C^{-2} where, as you see, the small subscripts are still retained for the dimensions, which are less often expressed than the numbers themselves.

Units and Constants

Base SI units have been used throughout – there are some constants that are usually expressed using dm^3 or cm^2 or Barns, or other non-primary units: I have avoided these, and everywhere converted them to using just metres, kilograms and seconds (etc.). The values of the constants I have used, together with the symbols used in calculation, are in the following table.

Name	**Sym.**	**Value**	**Units**	**Description and Source**
Theoretical speed of light in perfect vacuum	c	2.997924578E8	m/s	SI/BIPM definition [2]
Planck's Constant	h			[2]
Reduced Planck's Constant	$\hbar$	1.054E-34	J s	[2]
Avogadro Number		1.6605E-24		[2]
Planck Length	l_P	1.616199(97)E-35	m	[2]
parsec	p	1.542838790E18	m	the distance at which 1 AU subtends 1 second of circular arc [4]
classic electron radius		2.8179403267(27)E-15	m	[2]
Planck Time	t_P	5.39106(32)E-44	s	[2]
Ideal gas constant	R	8.314	J /K /mol	[2]
Astronomical Unit	AU	1.49597870700E8	m	[4]
Boltzmann Constant	k_B	1.380E-23	J /K	[2]
Atomic Mass Unit	m_u	1.6605E-27	kg	[2]

References

[1] **Kelly, I. D. K.;** *Dimensions of the Universe – I*, 2015, privately available.

[2] http://physics.nist.gov/constants inspected 2015-03-30.

[3] **Ruoff, Aurthur L., Ghandehari, Kouros;** *The Refractive Index of Hydrogen as a Function of Pressure*, Modern Physics Letters B, 07, 907 (1993). ISSN 0217-9849 (See http://www.world-scientific.com/doi/abs/10.1142/S0217984993000904)

[4] http://en.wikipedia.org/wiki/Parsec (examined 2015-04-02) which refers to International Astronomical Union, ed. (31 August 2012), *RESOLUTION B2 on the re-definition of the astronomical unit of length.*

[5] http://en.wikipedia.org/wiki/Neutrino (examined 2015-04-02), which contains numerous cross-references, in particular Anicin, Ivan C.; The Neutrino, Its Past, Present and Future, available from arXiv:physics/0503172.

[6] http://en.wikipedia.org/wiki/Planck_units (examined 2015-04-02), which refers to "CODATA Value: Planck constant over 2 pi". *The NIST Reference on Constants, Units, and Uncertainty*. US National Institute of Standards and Technology. (2011).

[7] http://en.wikipedia.org/wiki/Real_gas (examined 2015-04-02).

[8] http://en.wikipedia.org/wiki/Redlich%E2%80%93Kwong_equation_of_state (examined 2015-04-02), which refers to **Redlich, Otto; Kwong, J. N. S.** (1949). "On The Thermodynamics of Solutions". *Chem. Rev.* **44** (1): 233–244. doi:10.1021/cr60137a013. PMID 18125401 and **Redlich, Otto** (1975). "On the Three-Parameter Representation of the Equation of State". *Industrial & Engineering Chemistry Fundamentals* **14** (3): 257–260. doi:10.1021/i160055a020.

[9] http://en.wikipedia.org/wiki/Cosmic_microwave_background (examined 2015-04-02), which contains numerous further references.

[10] http://en.wikipedia.org/wiki/Photon (examined 2015-04-02), which contains further references.

[11] http://en.wikipedia.org/wiki/Global_Positioning_System (examined 2015-04-04), particularly the section #Satellite_frequencies

[12] http://en.wikipedia.org/wiki/Tests_of_general_relativity (examined 2015-04-04), particularly the section #Perihelion_precession_of_Mercury Note also the contained list of reference

[13] http://en.wikipedia.org/wiki/Metric_expansion_of_space (examined 2015-04-04)

[14] http://en.wikipedia.org/wiki/Accelerating_universe (examined 2015-04-04) Note that this discusses some of the alternative – and conflicting – suggestions for the degree, and the cause, of universal expansion

[15] http://en.wikipedia.org/wiki/Speed_of_light (examined 2015-04-04), particularly the section #Increased_accuracy_of_c_and_redefinition_of_the_metre_and_second

[16] http://en.wikipedia.org/wiki/Atomic_clock (examined 2015-04-04), particularly the section #Evaluated_accuracy

[17] http://en.wikipedia.org/wiki/Classical_electron_radius (examined 2015-04-04), referring on to http://www.cstl.nist.gov/div837/837.02/epq/Version1/doc/gov/nist/microanalysis/EPQLibrary/PhysicalConstants.html which refers on to http://physics.nist.gov/cuu/Constants/Table/allascii.txt for the

base values

[18] https://www.physicsforums.com/threads/refractive-index-by-pressure-temperature.762885/ (examined 2015-04-10), which computes the refractive index of H2 at 2.7° K and 5E-18 Pa pressure as $1+\epsilon$ where $\epsilon = 4.9354E\text{-}23$