

# Light Speed Uncertainty III

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## 0. Notation

- $\epsilon_0$  Vacuum permittivity
- $\mu_0$  Vacuum permeability
- $q_i$  Modulus electric charge on fermion type  $i$
- $e$  Modulus of electron charge
- $G$  Gravitational constant
- $N$  Numerical density of virtual pair
- $\sigma$  Cross-section area for photon capture
- $c_0$  Bare photon velocity  $c_0 \gg c$  (possibly very much greater than  $c$ )
- $c_\infty$  abstract velocity of light infinitely far from all mass at 0 K , perhaps the same as  $c$
- $c$  Einstein limit – normally taken to be the free vacuum speed of light at 0 K
- $c_{rel}$  Maximum light-speed in context, as permitted by the Lorentz Transformation
- $\phi$  Actual transmission velocity of light (non-theoretical) in some specific physical situation

$$r_s = \frac{2GM}{c^2} \quad \text{the Schwarzschild radius}$$

- $\lambda$  generalized wavelength
- $\lambda_0$  wavelength in free vacuum
- $H_0$  Hubble Constant (HC) in *km/s/megaparsec*
- $H_u$  Hubble Uncertainty

$$H_m = \frac{H_0}{M_m} \quad \text{Hubble Constant in } m/s/m$$

- $M_m$  Metres in a megaparsec  $3.085677581 E+22 m$

- $\lambda_{H_\alpha}$  Hydrogen alpha wavelength
- $f_{H_\alpha}$  Hydrogen alpha frequency
- $h$  Planck's Constant
- $\hbar$  Reduced Planck Constant
- $m_e$  rest mass of electron
- $n$  refractive index
- $R$  Radius of observable space

## 1. Introduction

### 1.1 Summary

This paper considers some aspects of the measurement of, and statements of the speed of light. It is the third in a series of papers: see [KELL16] and [KELL17] for the first two of the series. This paper looks in more detail at the possible consequences of light travel speed  $\phi$  always being less than Einstein's Limit  $c$ , and it makes some suggestions as to further theoretical and practical, experimental work.

## 1.2 Hypotheses

Some of these hypotheses and conclusions will be further explained in the body of this paper (and were discussed in both [KELL16] and [KELL17]), but they are all – more or less – heretical in the standard interpretation.

- $\phi$  (actual light speed) is *always* less than  $c$  (Einstein's Limit) [KELL16];
- actual light-speed  $\phi$  being less than the limit  $c$  does not affect the bulk of existing theories over short distances and across short times [KELL16];
- There is nowhere in the universe where light-speed  $\phi$  is actually  $c$  (Einstein's Limit) [KELL16] [KELL17];

## 2. Refractive Index

### 2.1 Definition

The refractive index of a medium is normally defined as the relationship of the velocity of light passing through that medium, and the fastest possible speed of light in free vacuum, Einstein's Limit,  $c$ . Thus if  $v$  is the phase velocity of light passing through some given medium, then its refractive index  $n$  is given by

$$n = \frac{c}{v}$$

The refractive index is a dimensionless number. It can never be less than one, as that would imply a transfer at faster than  $c$ , Einstein's Limit<sup>1</sup>

Let the refractive index  $n$  be represented by  $n = (1 + \rho)$  where  $\rho$  is the deviation from unity being considered. We discuss here the value of  $\rho$  arising from (i) the presence of matter interacting directly with the photons of the light (*e.g.* electron absorption and re-emission), and (ii) the gravitational effect upon the structure of the environment, even in the absence of direct interaction of a photon with absorbing matter, and (iii) the frequency  $\lambda$  of the light. We start with the standard absorption-emission [not by ephemeral fermions] part of the refractive under consideration.

### 2.2 Dependence Upon Wavelength

Refractive index depends upon the frequency of the electromagnetic radiation being refracted. Note the formula [FIBE17]:

$$n(\lambda) = \frac{\lambda_0}{\lambda} \quad \{2.1\}$$

which relates refractive index with the wavelength of the light in the medium,  $\lambda$ , and with the free vacuum wavelength  $\lambda_0$ . This formula applies across the full frequency range of electromagnetic radiation, and not just across some limited range.

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<sup>1</sup> Note that there are situations in which it is convenient to use in computation a refractive index less than one, but these situations do not imply the possible transfer of information between the source and destination of the light at faster than  $c$ , and these are situations which are not in fact relevant to the free transmission of light in space.

When considering the refractive index for visible light, only one tiny part of the whole possible spectrum is being looked at. When the global speed of electromagnetic radiation is being measured then the refractive index for all of the relevant frequencies / wavelengths must be considered. There is one range of wavelengths that is of particular interest here, which are those waves of the longest frequency and longest wavelengths, for it is in this range that the CMWB lies, and what may come beyond it, as time passes. The name for that range given in the following table, Maximally Low Frequency, is not to be taken as an assertion that no lower frequencies are possible, but merely as the name of a range.

Class		Frequency $f$	Wavelength $\lambda$	Energy	
Ionizing radiation	$\gamma$	Gamma rays	300 EHz	1 pm	1.24 MeV
	HX	Hard X-rays	30 EHz	10 pm	124 keV
	SX	Soft X-rays	3 EHz	100 pm	12.4 keV
			300 PHz	1 nm	1.24 keV
	EUV	Extreme ultraviolet	30 PHz	10 nm	124 eV
Ultraviolet	NUV	Near ultraviolet	3 PHz	100 nm	12.4 eV
Visible	NIR	Near infrared	300 THz	1 $\mu\text{m}$	1.24 eV
Infrared	MIR	Mid infrared	30 THz	10 $\mu\text{m}$	124 meV
	FIR	Far infrared	3 THz	100 $\mu\text{m}$	12.4 meV
Micro-waves and radio waves	EHF	Extremely high frequency	300 GHz	1 mm	1.24 meV
	SHF	Super high frequency	30 GHz	1 cm	124 $\mu\text{eV}$
	UHF	Ultra high frequency	3 GHz	1 dm	12.4 $\mu\text{eV}$
	VHF	Very high frequency	300 MHz	1 m	1.24 $\mu\text{eV}$
	HF	High frequency	30 MHz	10 m	124 neV
	MF	Medium frequency	3 MHz	100 m	12.4 neV
	LF	Low frequency	300 kHz	1 km	1.24 neV
	VLF	Very low frequency	30 kHz	10 km	124 peV
	ULF	Ultra low frequency	3 kHz	100 km	12.4 peV
	SLF	Super low frequency	300 Hz	1 Mm	1.24 peV
	ELF	Extremely low frequency	30 Hz	10 Mm	124 feV
			3 Hz	100 Mm	12.4 feV
Cosmic Background	GLF	Giga low frequency	0.3 Hz	1 Gm	1.24 feV
	CLF	Cosmic low frequency	0.03 Hz	10 Gm	0.124 feV
	MLF	Maximally low frequency	0.003 Hz	100 Gm	0.0124 feV

Frequency and wavelength are normally expressed as being related by

$$c = \frac{f}{\lambda}$$

Note that there are two representations of this formula: the usual one being in terms of the free

vacuum frequency  $f_0$  and the free vacuum wavelength  $\lambda_0$  :

$$c = \frac{f_0}{\lambda_0}$$

and the other in terms of the local actual speed of light transmission,  $\phi$  where  $\phi < c$  , and the local frequency,  $f$  , and the local wavelength,  $\lambda$  :

$$\phi = \frac{f}{\lambda}$$

The refractive indices of a number of free space environments need to be considered, covering all the classes from  $\gamma$  to MLF – and possibly beyond at each end. There is no need to consider here in detail local planetary environments, where the refractive indexes are directly observable. The environments that must be considered here must include:

- free space close to stars
- free space within a planetary system
- free space between stars, within a galaxy
- free space between galaxies in the same group
- free space between galaxy groups.

For some of these it is easy to accept, without further examination, a non-zero value for  $n(\lambda)$  for all values of  $\lambda$  : these are already known about, for example, for the atmosphere (and general environment) of the Sun, and the presence of interplanetary gas. Each of these has a perceptible refractive index greater than one. In subsequent sections wavelengths will be brought into the calculations.

### 2.3 Dependence Upon Temperature & Pressure

Refractive index through a gas is related to both the temperature and pressure of that gas. Necessary considerations include the substance of a gas (“what form of gas it is” – diatomic hydrogen, deuterium, helium, free particles, etc.),  $S$ , and the Temperature,  $T$  in Kelvin, the pressure,  $p$  in Pascals, and the wavelength,  $\lambda$  in metres, giving an abstract formula for refractive index:

$$n_f = f(S, T, p, \lambda)$$

This formula is not a simple one, and will depend upon many characteristics of the gas. Its value gives the increment up one of the refractive index upon the given parameters. If only one sort of matter (for example, monatomic hydrogen) is being considered, where  $S$  is fixed, then a simplified formula can be used:

$$n_{sf} = f(T, p, \lambda)$$

(Note that this is not a simplification we can use in the full calculation, and we do eventually need to consider the full  $n_f$  .) This function  $n_{sf}$  is a function over three variables, and can be empirically researched plane by plane – for constant pressure, for constant temperature and for constant wavelength.

Here we consider just three environments from our list: between stars, between galaxies, and between groups. For each of these environments we must to consider temperature, pressure and substances present, and estimate the refractive indexes over a range of wavelengths.

A base formula, from [FIBE17] gives a relation between the light frequency within the medium,  $\lambda$  , with the base frequency in vacuum  $\lambda_0$  :

$$n(\lambda) = \frac{\lambda_0}{\lambda} \tag{2.1}$$

which implies that the phase velocity of a burst of light containing just a narrow range of frequencies is

$$v_p = \frac{c}{n(\lambda)} \quad \{2.2\}$$

There are several formulae which attempt to relate refractive index with specific gravity<sup>2</sup>, but these are not used here, as we are considering a very tenuous gas.

For each of these densities, and the relevant mixtures of substances, we need the ability to calculate and measure the corresponding refractive indices. Different regions of free space have different proportions of molecular hydrogen, free protons and electrons and neutrons, molecular helium, alpha particles, and (in small proportion) all the other elements, some of which will be as more concentrated particulate fragments (gas in the ordinary sense, and dust). In the calculations here, there is no consideration made of dark matter or dark energy – that is something to be done elsewhere.

NOTE:

## 4. Measurement of Light-Speed

### 4.1 Experimental Measurement

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## 5. Photon Distribution

### 5.1 Photon Decay

Any given photon will, over time, lose energy. This energy is passed on to other entities in the universe – to other subatomic particles. If this were not so, then the universal quantity of high-energy photons – photons as, say, they are radiated from stars, as cosmic rays, X rays, light, etc. – would continually increase. Such an increase is not observed, but we *do* observe the CMWB, which has photons of very low frequency and very long wavelength – hence of low mass/energy. The CMWB appears to fill the whole of the observable universe [an experimental observation], and may also be a weak example of the ultimate path of every photon [an illustrative supposition].

Each photon, therefore, gradually decreases in frequency and increases in wavelength. No photon has yet been observed that has zero frequency and infinite wavelength: this may be because the universe is not yet that old (no photon has decayed that far) or because there is some limit in the universe which prevents that extreme of decay. Some possible hypotheses could be looked at – but all of these are purely speculative – such as:

- The maximum wavelength is the current diameter of the universe (if that term has a real meaning). Note that this is different from the observable diameter,  $2R$
- The minimum frequency is one cycle in the current age of the universe (which is equally difficult to define)

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<sup>2</sup> For example, Gladstone and Dale, Lorentz-Lorenz, Lichtenecker, Clausius-Mossetti, *et al.*

- There is no maximum wavelength or minimum frequency, but the extreme case (infinite wavelength, zero frequency) is approached asymptotically over the continued existence of the universe, but has never yet actually been achieved
- There will be at some place and at some time at least one photon which has zero energy. The consequences of this are difficult [for me!] to deduce.

## 5.2 Number of Photons

How much light is there in the universe?

This is too imprecise a question to answer. Rather something much more detailed should be asked, such as: for each electromagnetic frequency/wavelength, what is the universal quantity of that sort of radiation, and how is that radiation distributed in both time and space?

The quantity of matter in the universe seems to be close to the critical density to make the overall curvature of the universe zero. This density is given by

$$\rho_c = \frac{3H_0^2}{8\pi G} \quad \{5.1\}$$

where  $H_0$  is the Hubble Constant,  $G$  is the Gravitational Constant, and  $\rho_c$  is the critical density [WIKI16E]. From the European Space Agency's Planck Telescope results, this come to  $8.5 \text{ E-}26 \text{ kg/m}^3$ , which is commonly quoted as about 5 hydrogen atoms per cubic meter. This estimate is then usually analysed as consisting of about 4.8% ordinary matter, 0.1% neutrinos (together making just 4.9%), 26.8% cold dark matter, and 68.3% dark energy. Neutrinos are very difficult to observe, and both their individual and total mass are extremely difficult to measure – thus there is a strong possibility that the mass of the neutrinos is rather greater than 0.1%.

Dark Matter and Dark Energy are, outside of the scientific community, intuitively very non-obvious – which does not, in any way, prove that they do not exist – but it raises questions as to whether the analysis that requires them has missed some other sources of gravitational matter. If electromagnetic radiation, in most of its forms, (1) can be seen to be expressing some of its mass/energy as mass, and (2) there is sufficient electromagnetic radiation, and (3) the sum amount of this mass (distributed according to the distribution of the radiation) is sufficient to cover the 26.8%+68.3% or 95.1% of the total mass, then it would be possible to remove these two non-obvious (and very large) contributions to the apparent mass of the universe. This is a big “if”, and would have to be very carefully justified.

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## 5.3 Total Photon Mass/Energy

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## 6. Units and Testing

### 6.1 Units

None of the arguments above depend deeply upon the type of definition being used for the metre

distance unit – if the arguments are changed back to using the current, SI, definition of the metre, then, provided light has always travelled at its currently observed rate over proper distance, the suggestions made here are not affected. The assumption is that the definition of the metre may be projected back in time, right back to the Big Bang (or other stated base point).

None of the arguments above depend deeply upon the type of definition being used for the time second unit, provided that definition can be projected back in time, right back to the Big Bang (or other stated base point).

The fundamental suggestion being made here is that the Hubble Expansion takes place between *all* spatial points in the universe, whether or not there is bound matter between those points.

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## 6.2 Experimental Testing

None of these assumptions have been experimentally tested. How they can be tested is not something for which the experiments can be usefully outlined here – other workers in this field will need to find experimental tests. Note, however, that entirely *theoretical* tests are insufficient: the suggestions being made here are in minor contradiction to some of the base assumptions made in the current theoretical analyses of Quantum and Astrophysical and Relativistic theories. This is not saying that any of these branches of theoretical analysis are wrong – merely that they are currently insufficient, and that a very few changes in base assumptions may be able to bring them closer together.

Some theoretical questions can be posed and analyses can be made, however. These should include:

- Experimentation giving a more precise knowledge of the value of the Hubble Constant, whose value is currently known with very limited precision
- Examination as to whether, and by how much, the Hubble Constant has changed over time
- Examination of the theoretical consequences of a truly universal Hubble Expansion, even when that takes place at a sub-atomic level
- Examination of what the number of neutrinos could be in various parts of space, and of what their total mass could be
- Examination of what definition of distance (what type of distance) is actually used when (a) measuring, and (b) stating the speed of light. These should be the same type of distance in all cases, but currently may well not be
- Theoretical examination of what might be the actual speed of light, under varying conditions within the universe (remembering that  $\phi$  is always less than  $c$ ), and of the distribution of these various light speeds
- Theoretical examination of what “the speed of light” means in each place that it is used in theory – whether it is Einstein’s Limit  $c$  or whether it is the actual local speed of light  $\phi$  which will always be different (by small or large amount) from  $c$
- Theoretically estimating the total quantity of light in the universe, how that light is distributed, and what the spread of frequencies is for that light. Note that some work has already been done on this, when considering the CMWB, but it could be suggested that there may be a lot more light that is currently allowed for, and that the consequent gravitational effect of that light is not fully been taken into account (*i.e.* “dark matter”

and “dark energy” may actually be the electromagnetic radiation (light, etc.) that is present)

- Theoretical estimation of the mass contribution required by all existing electromagnetic radiation to allow for the combined masses of “dark matter” and “dark energy”, and verification that this mass contribution could be distributed so that this could provide all the extra apparent gravitational mass of galaxies.

Some practical experimentation:

- Practical experiments would have to show whether or not the Hubble Expansion takes place even for physically connected locations
- Practical, as well as theoretical, experiments are needed to give a more precise estimate of the radius of the observable universe
- Practical experiments would have to show, with more certainty, both what is the number and density of neutrinos in the universe, and also what is their individual and total mass
- Practical, experimental evidence should give support for any improved theoretical estimates of the total quantity of light in the universe.

One possible test for photons having mass might be to shine two light beams at right angles to each other, so that they very nearly cross each other. Then if there is attraction of one beam to the other, then the path of one beam will be affected by whether the second beam is on or not.

## 7. Possible Consequences

It seems that each individual photon both loses and gains energy, depending upon its trajectory, but will in general, after an extremely long time, dissipate all of the energy that it can lose whilst still remaining in existence. The CMWB shows how far this decay has gone for the vast number of photons that fill the universe [FIXS09] [WIKI16].

If actual light speed,  $\phi$ , is always smaller than the Einstein Limit,  $c$ , then it would be possible for the photon to have a rest mass. This would be possible only if (a) a photon, whilst still in existence, can come to rest, and (b) the rest mass is small enough to avoid an impossibly large mass when travelling at  $\phi$ , which is close in value to  $c$ . There is, however, no experimental evidence that a photon can actually come to rest whilst still containing mass/energy. The current view of photons having no rest mass is one which can still be maintained under the suggestions made in this paper. But the mass/energy of such photons as do exist cannot be ignored in gravitational and density calculations.

There are several sources of difference between the magnitudes of  $\phi$  and  $c$ :

- Hubble Expansion being truly universal
- Non-zero gravitational state everywhere in the universe
- Refractive index always greater than one.

This paper has considered only the third of these. There should be consideration elsewhere of the overall effect of the other two sources of difference.

The enormous number of photons in the universe contribute hugely to its mass/energy. What is not clear is just how many photons there are in the universe, and what the distribution is for their frequencies and wavelengths.

## 8. Physical Values Used

Symbol	Meaning	Value
$c$	Einstein's Limit; the theoretical upper limit for the speed of light in vacuum	$2.99792458 E+8 \text{ m/s}$ [by SI definition]
$\phi$	Actual local speed of light	Variable, according to local conditions
$H_0$	Hubble Constant	$70 \text{ km/s/Mpc}$ [approximate measurement – rather uncertain]
$H_\alpha$	Reduced Hubble Constant	$2.2685 E-18 \text{ m/s/m}$ [calculated from $H_0$ ]
$H_u$	Hubble Uncertainty	1.32154 in $1.0 E+26$ [calculated from $H_0$ and $c$ ]
T	Clock tick, caesium 133	9 192 631 770 [by SI definition]
R	Radius of observable space	$4.4 E+26 \text{ m}$ [estimate – very uncertain]

Note that, apart from those values set by definition, rather than observation, all of these values used in our preliminary calculations here are rather uncertain. This means that most of these, and subsequent, calculations are, themselves, uncertain.

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Once objects are bound by gravity, they no longer recede from each other. Thus, the Andromeda galaxy, which is bound to the Milky Way galaxy, is actually falling *towards* us and is not expanding away. Within the **Local Group**, the gravitational interactions have changed the inertial patterns of objects such that there is no cosmological expansion taking place. Once one goes beyond the Local Group, the inertial expansion is measurable, though systematic gravitational effects imply that

larger and larger parts of space will eventually fall out of the "Hubble Flow" and end up as bound, non-expanding objects up to the scales of **superclusters** of galaxies. We can predict such future events by knowing the precise way the Hubble Flow is changing as well as the masses of the objects to which we are being gravitationally pulled. Currently, the Local Group is being gravitationally pulled towards either the **Shapley Supercluster** or the "Great Attractor" with which, if dark energy were not acting, we would eventually merge and no longer see expand away from us after such a time.

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There are many galaxies visible in telescopes with **red shift** numbers of 1.4 or higher. All of these are currently traveling away from us at speeds greater than the speed of light. Because the **Hubble parameter** is decreasing with time, there can actually be cases where a galaxy that is receding from us faster than light does manage to emit a signal which reaches us eventually. <sup>[23][24]</sup>

hhh <http://refractiveindex.info/?shelf=main&book=N2&page=Peck-0C>

<http://refractiveindex.info/?shelf=main&book=N2&page=Peck-0C>

*Atomic Data and Nuclear Data Tables* **14**, 21-37 (1974)

## Symbols Used

$\epsilon_0$  Vacuum permittivity

$\mu_0$  Vacuum permeability

$\mu_i$  Magnetic moment of pair type  $i$

$q_i$  Modulus electric charge on fermion type  $i$

$e$  Modulus of electron charge

$Q_i = q_i/e$

$\xi_i$  Average energy of fermion pair of type  $i$

$N_i$  Vacuum density of fermion pairs of type  $i$

$\lambda_i^C$  Compton wavelength of fermion of type  $i$

$\delta_i = K_\delta \frac{\lambda_i^C}{2} = \frac{K_\delta \hbar}{2 m_i c}$  where  $K_\delta \approx 1$

$N_i = \frac{2}{\delta_i^3}$  Vacuum density of fermion pairs of type  $i$

$c^2 = 1/\epsilon_0 \mu_0$

$N$  Numerical density of virtual pair

$\sigma$  Cross-section area for photon capture

$\Lambda = (\sigma N)^{-1}$  Mean free path between interactions

$c_0$  Bare photon velocity  $c_0 \gg c$  (very much greater than  $c$ )

$\Lambda/c_0 + \tau$  Mean time for a photon to cross length  $\Lambda$

$$v = \frac{\Lambda}{\frac{\Lambda}{c_0} + r} = \frac{1}{\frac{1}{c_0} + \sigma N_\tau} \quad \text{mean velocity for a photon}$$

as  $c_0 \rightarrow \infty$  we have  $v \rightarrow \frac{1}{\sigma N_\tau}$

$$\sigma_{Thomson} = \frac{8\pi}{3} \alpha^2 (\lambda^C)^2 \quad \text{where } \lambda^C = \hbar / (m_e c) \quad \text{the Compton wavelength of the electron}$$

(But since we have  $\alpha^2 = \alpha_1 \alpha_2$  and  $\alpha_2 = 1$  we can drop the squaring. This is the probability of a photon being absorbed – its probability of being emitted is 1 – it will certainly be emitted.

thus  $\sigma_i = K_\sigma \frac{8\pi}{3} \alpha Q_i^2 (\lambda_i^C)^2$  where  $K_\sigma \approx 1$

$$\frac{1}{\Lambda} = \sum_k \frac{1}{\Lambda_k} \quad \text{Mean free path over all fermion types, thus:}$$

$$\Lambda = \frac{1}{\sum_k \sigma_k N_k}$$

$$\text{Average velocity of a photon in vacuum } v = \frac{3 K_\delta^3 K_\xi}{K_\sigma 32 \pi \alpha \sum_i Q_i^2}$$

$$\text{If summed over all charged fermions we have } v = \frac{3 K_\delta^3 K_\xi}{K 256 \pi^{alpha}} = \frac{K_\delta^3 K_\xi}{K_\sigma} 0.51 c$$

therefore  $v = \frac{1}{K_\sigma} c$  so  $K_\sigma = 1$  iff  $v = c$

If we have a mass  $M$  at distance  $r$  then the refractive index  $n_r$  at that distance is given by

$$n_r = 1 + \frac{2GM}{r c_\infty^2} = 1 + \frac{R_s}{r} \quad \text{where } R_s = \frac{2GM}{c_\infty^2} \quad \text{is the Schwarzschild radius}$$

$$c_r = \frac{c_\infty}{n_r} = c_\infty \left(1 - \frac{R_s}{r}\right) \quad \text{So the presence of matter changes the refractive index of empty space –}$$

provided we allow the appearance-disappearance of fermion pairs: “ephemeral particle pairs” [URBA13].

$W_i$  Average energy of pair of type  $i$  (contrast with  $\xi_i$ )

$W_i = K_W 2 m_i c_{rel}^2$  where  $c_{rel}$  is the maximum velocity arising from the Lorentz transformation  
 $c_{rel}$  is not necessarily equal to  $c$

$$\text{Lifetime of a pair of type } i \text{ is } \tau_i = \frac{\hbar}{2 W_i} = \frac{1}{K_W} \frac{\hbar}{4 m_i c_{rel}^2}$$

One electron spin state occupies a hyper volume  $h^3$

$\Delta x_i$  is the spacing between identical  $i$ -type fermions, with  $p_i$  being their average momentum, then the one dimensional hyper volume is  $p_i \Delta x_i$

This gives  $p_i \Delta x_i / h = 1$  thus  $\Delta x_i = 2\pi \hbar / p_i$

$$\text{If relativity holds for the ephemeral pairs then } \Delta x_i = \frac{2\pi \hbar c_{rel}}{\sqrt{(W_i/2)^2 - (m_i c_{rel}^2)^2}} = \frac{\lambda_{C_i}}{\sqrt{K_W^2 - 1}}$$

where  $\lambda_{C_i}$  is the Compton length associated with fermion type  $i$  given by  $\lambda_{C_i} = \frac{h}{m_i c_{rel}}$

Pair density is  $N_i \approx \frac{1}{\Delta x_i^3} = \left( \frac{\sqrt{K_W^2 - 1}}{\lambda_{C_i}} \right)^3$

$$\mu_i = \frac{e Q_i \hbar}{2 m_i} = \frac{e Q_i c_{rel} \lambda_{C_i}}{4 \pi}$$

Vacuum permeability  $\tilde{\mu}_0 = \frac{K_W}{(K_W^2 - 1)^{3/2}} \frac{24 \pi^3 \hbar}{c_{rel} e^2 \sum_i Q_i^2}$

Since for the three families of the standard model we have  $\sum_i Q_i^2 = 8$  then

we have  $\tilde{\mu}_0 = \frac{K_W}{(K_W^2 - 1)^{3/2}} \frac{3 \pi^3 \hbar}{c_{rel} e^2}$

calculated permeability is equal to observed permeability  $\tilde{\mu}_0 = \mu_0$  when

$$\frac{K}{(K_W^2 - 1)^{3/2}} = \mu_0 \frac{c_{rel} e^2}{3 \pi^3 \hbar} = \frac{4}{3} \frac{\alpha}{\pi^2} \quad \text{which gives} \quad K_W \approx 31.9$$

vacuum permittivity  $\tilde{\epsilon}_0 = \frac{e^2}{6 \pi^2} \sum_i \frac{N_i Q_i^2 \lambda_{C_i}^2}{W_i}$  and with quoted value for  $K_W$  this gives

$$\tilde{\epsilon}_0 = 8.85 E - 12 F/m$$

mean free path of a photon between successive interactions  $\Lambda_i = \frac{1}{\sigma_i N_i}$

average photon velocity  $\bar{c} = \frac{1}{\sum_i \sigma_i N_i \tau_i / 2}$  which gives  $\bar{c} = \frac{K_W}{(K_W^2 - 1)^{3/2}} \frac{16 \pi}{\sum_i (\sigma_i / \lambda_{C_i}^2)} c_{rel}$

From [MONI03] the optical path difference over length  $l$ , matter density  $N$  (molecules per area, hence  $Nl$  is number of molecules between source and observer), and average polarizability of  $\alpha$  is  $\delta = 2 \pi N l \alpha$ . For a gas made only of H<sub>2</sub> molecules  $\alpha = 8.02 E - 25 cm^3$  (from CRC Handbook of Chemistry and Physics 79th Edition, 1988-1989)

[URBA03] Semi-classical formula for refractive index includes  $n^2 - 1 = 4 \pi r_e (\hbar c)^2 \sum_i \frac{N_i}{W_i^2 - E_y^2}$

where  $\sum_i N_i = N_{elec}$  where  $N_{elec}$  is the number of valence electrons per unit volume, and

$r_e$  is the classical radius of the electron  $2.83 E - 15 m$ , and this is the formula for a photon of energy  $E_y$  and the electrons in the molecule operate as discrete oscillators with energy  $W_i$ ,

$\hbar c = 197.3 MeVfm$ , the hydrogen atom has  $W_0 = -13.6 eV$  and  $W_1 = -3.5 eV$ . But there is

an alternative, which is the Maxwell-quantum formula:  $n^2 - 1 = 4 \pi r_e N_{elec} (\hbar c)^2 \sum_i \frac{f_i}{E_i^2 - E_y^2}$

where  $\sum_i f_i = 1$ . If the ground state of the atom is  $W_0$  then  $E_i = W_i - W_0$ . For the hydrogen atom we have  $E_1 = W_1 - W_0 = 10.2 eV$ . Therefore the Maxwell-quantum formula differs mainly in the characteristic energies  $E_i$  which are smaller than the  $W_i$  of the semi classical formula.

...But there are problems, as neither of these formulae have been properly checked against experimental results. And there are other sources of formulae for refractive index:

$$n_{Cauchy} = A + B\lambda^2 + C\lambda^4 \quad (1840), \quad n_{Sellmeier} = \sqrt{1 + \frac{B_1\lambda^2}{\lambda^2 - C_2} + \frac{B_2\lambda^2}{\lambda^2 - C_2}} \quad (1871), \text{ and}$$

$$n_{Hartmann} = A + \frac{B}{\lambda - \lambda_0} \quad (1900). \text{ So we seem to have no agreed, and experimentally tested, formula so far.}$$

Alternative formulation [URBA07] is: photon borrows energy  $\Delta E_i = E_i = E_y$  to be stopped, stays inside (the atom/electron) for time  $t_i = \frac{\hbar}{\Delta E_i}$ . Simplify this into a single average  $\langle \Delta E \rangle$  and time  $\langle t_{stop} \rangle$

and allow the re-emitted photon to travel to its next absorption-emission at speed  $c$  (the speed of light in vacuum), then the stop time is roughly  $c \langle t_{stop} \rangle = \frac{2000 eV \text{ \AA}}{10 eV} = 200 \text{ \AA}$  (a small number of Angstroms) If

we take the mean free path between collisions to be  $\Lambda$  then the total average time to cross distance  $\Lambda$  is  $\langle t_{stop} \rangle + \Lambda/c$  which gives the average velocity to be  $V = \Lambda (\langle t_{stop} \rangle + \Lambda/c)^{-1}$  and hence the

refractivity is given by  $n - 1 = \frac{c \langle t_{stop} \rangle}{\Lambda}$

[IDKK] Check what modifications are required to above formulae what the inter-absorption speed is not  $c$

but some formula such as  $c_r = \frac{c_\infty}{n_r} = c_\infty \left(1 - \frac{R_S}{r}\right)$  or  $\bar{c} = \frac{1}{\sum_i \sigma_i N_i \tau_i / 2}$  which depend upon the gravit-

ational effects of nearby matter – and “nearby” here means all the matter within gravitational visibility of the

moving photon. In fact the formula  $c_r = \frac{c_\infty}{n_r} = c_\infty \left(1 - \frac{R_S}{r}\right)$  could be expanded to

$c_{local} = c_\infty \left(1 - \sum_j \frac{R_{Sj}}{r_j}\right)$  to cover all gravitational bodies, each at distance  $r_j$  from the photon being considered

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13

$$m(r)c^2(r) = m_\infty c^2$$

$$\frac{GMm_\infty}{r} = m_\infty c^2$$

$$1 - \frac{R_s}{2r}$$

$$m(r)c^2(r) = m_\infty c_\infty^2 - \frac{GMm_\infty}{r} = m_\infty c_\infty^2 \left(1 + \frac{R_s}{2r}\right)$$

$$\alpha = \frac{e^2}{(4\pi\epsilon_0\hbar c)} \text{ is a constant}$$

$$\tau(r) = \frac{\hbar/2}{K_\xi 2m(r)c^2(r)} = \frac{\hbar/2}{K_\xi 2m_\infty c_\infty^2 \left(1 - \frac{R_s}{2r}\right)} = \tau_\infty \left(1 - \frac{R_s}{2r}\right)^{-1} \text{ and where } r \gg R_s \text{ this approx-}$$

$$\text{imates to } \tau(r) \approx \tau_\infty \left(1 + \frac{R_s}{2r}\right)$$

$$\frac{\Delta c}{c} = \frac{2GM_\odot}{Rc_\infty^2} \approx 6E-10$$

$$\frac{\Delta c}{c} = \frac{2GM_\odot}{Rc_\infty^2} \approx 6E-10$$

$$\frac{\Delta \epsilon_0}{\epsilon} = \frac{\Delta \mu_0}{\mu} = \frac{2GM_\odot}{Rc_\infty^2} \frac{\Delta R}{R} \text{ where } M_\odot \text{ is the mass of the Sun}$$

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